A Mathematical Modeling Approach and a Novel Solution for Sector Azimuth Angle Planning

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***Abstract*—State of the art planning tools rely on intricate simulation models and metaheuristic approaches to determine the optimal azimuth angle of each sector in a system. The process is highly time consuming and the quality of outcome is heavily dependent on the practitioner's experience in setting input parameters. To address this problem, in this paper we exploit a mathematical approach and develop a system model that ultimately helps to obtain a low complexity analytical solution. In addition to providing insights into system behaviour as function of azimuth angles, the potential performance gain of the proposed optimisation solution is investigated through extensive numerical results.**

I. INTRODUCTION

Azimuth orientation of sectors casts a major impact on the cellular system performance as it determines the interference as well as coverage. However, in a plethora of academic works on cell planning, sectorization is mostly overlooked altogether [1]–[5] for sake of simplicity by assuming omnidirectional antennas. Some works on cell planing that do incorporate sectorisation, very often assume an ideally symmetric sector orientations, e.g. $[0^o, 120^o, 240^o]$ [6]. In commercial cellular systems, on the other hand, sector azimuth angle optimisation is a key component of the planning process and once Base Station (BS) locations are decided, azimuth angle have to be set optimally for each sector. This sector optimisation needs to take into account not only the antenna heights and orientations, but also the demographic and topographic factors.

Given the commercial significance, there are quite a few works that have addressed sector optimisation as part of cell planning process [7]–[14]. However, due to the non polynomial hard nature and unfathomable search space of the planning problem [15], metaheuristics such as simulated annealing [7], genetic algorithms [8], [9], particle swarm [10], [11], Taugchi's method [12], aunt colony optimisation [13], or k-mean algorithm [14] have been applied in these works to obtain solutions.

The basic methodology that has been generally followed in these works involves building a detailed simulation model that can predict the Key Performance Indicators (KPIs) as function of planning parameteres of interest. An acceptable solution is then searched by partially exploring the solution space via simulated evaluation of KPIs at parameter combinations selected by the aforementioned heuristic. The key advantage of this *simulation-heuristics* approach is that it allows to consider a large number of planning parameteres simultaneously in the optimisation process by capturing them in the system level simulation model. However, this approach suffers from the following drawbacks: 1) Building the simulation model and its calibration is a time consuming task; 2) Large computational times and memory space are generally required to obtain acceptable solution 3) the outcome of this approach heavily depends on the practitioners experience, i.e., the choice of the parametere used to initially configure the metaheuristic technique, largely determines the quality of obtained solution. For example, in case of simulated annealing–that is a widely used metaheuristic in cell planning–there are no general rules to set the *temperature* and *acceptance probability* relationships [16]. A detailed discussion on the use of metaheuristic in such problems can be found in [17]. Here it would suffice to comment that despite the prevailing use of *simulation-heuristics* approaches in cell planning problems, the quality of the solutions produced by them may not be asserted.

In this paper we exploit an alternative approach, i.e., instead of relying on the black box of an intricate simulation model, we build a more transparent analytical model to link the system level performance with the planning parameters that in turn allows appropriate mathematical tools to be invoked yielding more robust and efficient solutions. However, due to the high complexity emerging from the involvement of an amalgam of KPIs like coverage, capacity, and QoS and myriad of planning parameters such as BS location, tilt, azimuth, frequency, heights, transmission powers etc, the works that leverage a purely mathematical approach to solve planning problem are scarce in the literature. A possible remedy to overcome this difficulty as advocated in [18] is to exploit the old rule of divide and conquer, i.e., one or a few parameters at a time can be considered in the mathematical model and optimisation process. This strategy of divide and conquer has to be used often even in the *simulation-heuristic* based approach to overcome the complexity of the planning problem. For example, although the authors in [12] propose a *simulation+heuristic* based solution for planning tilt, azimuth and uplink power control parameteres, all three parametere are considered independent of each other using Taugchi method based metaheuristic. Alternatively, if a mathematical model can be developed that can be used to optimise these three parameters separately and solutions with similar or better performance can be obtained, the added advantages would be time-resource efficiency, tractability and better assurance

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$$\gamma_q^{n,m}(t) = \frac{P^{n,m} G_q^n \alpha \left(d_q^n\right)^{-\beta} \sigma_q^n \epsilon_q^{n,m}(t)}{N_o B + \sum_{\forall \acute{n} \in \mathcal{N} \setminus n} \sum_{\forall \acute{m} \in \mathcal{M}} \left(P^{\acute{n},m} G_q^{\acute{n}} \alpha \left(d_q^{\acute{n}}\right)^{-\beta} \sigma_q^{\acute{n}} . u\left(\mu_q^{\acute{n},\acute{m}}\right) \epsilon_q^{n,m}(t)\right)} \tag{1}$$

of quality of solution. In this paper, we pursue this objective and develop an analytical model that can be used to optimise a KPI of average spectral efficiency as function of sector azimuth angle, while taking into account the realistic non uniform user demographic distributions. Although there are some works that focus solely on sector optimisation [19]–[22], these works investigate optimal number of sectors for different objectives and do not investigate the optimal azimuth for given number of sectors and given user distribution. To the best of our knowledge, this paper is first to provide an analytical framework and solution of this kind for sector azimuth angle optimisation.

The rest of this paper is organised as follows. In section II we present system model, assumptions and problem formulation. In section III we present our proposed solution. Section IV is devoted to discussion of numerical results and key insights obtained and Section V finally concludes this paper.

II. System Model and Problem Formulation

Let $\mathcal{N}$ denote the set of points corresponding to the transmission antenna locations of all sectors. We assume that the total area of interest A is divided into set of bins denoted by $\mathcal{Q}$ such that $\sum_{i=1}^{Q} q_i = A$, and $\frac{A}{Q} = q, \forall q \in \mathcal{Q}$ where $Q = |\mathcal{Q}|$. Area q is so small that long term propagation conditions such as pathloss and shadowing can be assumed to be constant within q. With this assumption, the Signal to Interference and Noise Ratio i.e. SINR perceived on the m^{th} subcarrier in q^{th} bin being served by n^{th} sector at time t can be given by (1)

where $P^{n,m}$ and $P^{\acute{n},m}$ are the transmission powers from the n^{th} and $\acute{n}^{th}$ sector on m^{th} subcarrier, d_q^n and $d_q^{\acute{n}}$ are distances of q^{th} bin from n^{th} and $\acute{n}^{th}$ sector respectively. G_q^n and $G_q^{\acute{n}}$ in (1) are the antenna gains from the n^{th} and $\acute{n}^{th}$ sector to q^{th} bin. α and β are pathloss model coefficient and exponents respectively. N_o is thermal noise spectral density and B is carrier bandwidth and $u\left(\mu_q^{m,\acute{m}}\right)$ is unit function defined as:

$$u\left(\mu_q^{m,\acute{m}}\right) = \begin{cases} 1 & \acute{m} = m \\ 0 & \acute{m} \neq m \end{cases} \tag{2}$$

σ_q^n and $\sigma_q^{\acute{n}}$ are shadowing values that q^{th} bin faces while receiving signal from n^{th} and $\acute{n}^{th}$ sector, respectively. $\epsilon_q^{n,m}(t)$ is the fast fading coefficient for m^{th} subcarrier at time t. Since planning needs to take into account only the long term statistics of the prorogation conditions, therefore short term channel variations such as fast fading can be omitted as its generally compensated for by power margins. Clutter dependent shadowing however can have long term impact and cannot be neglected in the planning process. Also, in the planning process a worst case scenario of full system load is typically assumed. With this full load assumption, i.e., 100% subcarrier utilisation and a frequency reuse of one that is typical to LTE, the term $u\left(\mu_q^{m,\acute{m}}\right)$ will always be 1. Another implication of full load and frequency reuse of 1 is that downlink is generally interference limited. Furthermore, we assume power allocation across all subcarriers to be equal over long term. With these assumptions, for planning purposes, the SINR in (1) can be approximated with time and sub-carrier independent geometric SIR in (3), that still captures all the parameter that affect the planning process:

$$\gamma_q^n = \frac{P^{n,m} G_q^n \alpha \left(d_q^n\right)^{-\beta} \sigma_q^n}{\sum_{\forall \acute{n} \in \mathcal{N} \setminus n} \left(P^{\acute{n},m} G_q^{\acute{n}} \alpha \left(d_q^{\acute{n}}\right)^{-\beta} \sigma_q^{\acute{n}} \right)} \tag{3}$$

For 3GPP LTE and LTE-A, the three dimensional antenna gain can be modelled as (in dB):

$$G_q^n = \lambda_v \left(G_{max} - \min\left(12 \left(\frac{\theta_q^n - \theta_t^n}{B_v} \right)^2, A_{max} \right) \right) + \lambda_h \left(G_{max} - \min\left(12 \left(\frac{\phi_q^n - \phi_a^n}{B_h} \right)^2, A_{max} \right) \right) \tag{4}$$

where θ_q^n is the vertical angle in degrees from q^{th} bin to n^{th} sector and θ_t^n is the tilt angle of the n^{th} sector with respect to horizon. The ϕ_q^n is horizontal angle in degrees with similar meanings of subscript and postscript. Subscripts h, a and v denote horizontal, azimuth and vertical respectively. Thus B_h and B_v represent horizontal and vertical beamwidths of the antenna respectively, and λ_h and λ_v represent weighting factors for the horizontal and vertical beam pattern of the antenna in 3D antenna model respectively. G_{max} and A_{max} denote the maximum antenna gain at the boresight of the antenna and maximum antenna attenuation at the sides and back of the boresight of the antenna respectively, in dB. For simplicity of expression we can omit the maximum attenuation factor A_{max} in (4) and assume maximum gain of 0 dB for the time being. Though, these assumptions preserve the accuracy of the antenna model essential to the analysis in this paper, i.e., the parabolic dependency of the antenna model on the angle from the foresight is preserved; when evaluating the numerical results in Section IV, we will use actual values for G_{max} and A_{max}. The simplified antenna gain model in linear form can be written as:

$$G_q^n = 10^{-1.2\left(\lambda_v \left(\frac{\theta_q^n - \theta_t^n}{B_v}\right)^2 + \lambda_h \left(\frac{\phi_q^n - \phi_a^n}{B_h}\right)^2\right)} \tag{5}$$

For ease of expression we use the following substitutions:

$$\rho_k^n = \sigma_q^n \alpha \left(d_k^n\right)^{-\beta}; \varphi_q^n = \frac{B_h^2 \lambda_v}{\lambda_h} \left(\frac{\theta_q^n - \theta_t^n}{B_v} \right)^2; \tau = \frac{-1.2\lambda_h}{B_h^2} \tag{6}$$

Using (5) and (6), (3) can be written as:

$$\gamma_q^n = \frac{\rho_q^n 10^{\tau\left(\varphi_q^n + \left(\phi_q^n - \phi_a^n\right)^2\right)}}{\sum_{\forall \acute{n} \in \mathcal{N} \setminus n} \left(\rho_q^{\acute{n}} 10^{\tau\left(\varphi_q^{\acute{n}} + \left(\phi_q^{\acute{n}} - \phi_q^{\acute{n}}\right)^2\right)} \right)} \tag{7}$$

Note that (7) provides a model for estimating SIR and thus eventually system KPIs as a function of sector azimuth angles

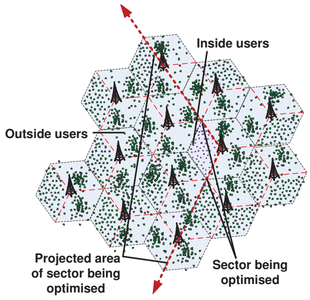

Fig. 1. System model

in the system. Now we focus on part of the planning process that this paper aims to tackle, i.e., finding the optimal azimuth angles $\tilde{\phi}_a^N$ in the system that maximise system wide user spectral efficiency for given user demography. Mathematically our problem can be written as:

$$\tilde{\phi}_a^N = \arg\max_{\phi_a^N} \sum_{\forall q \in \mathcal{Q}} w_q \log_2 \left(1 + \gamma_q \left(\phi_a^N\right)\right) \tag{8}$$

Here γ_q is SIR in q^{th} bin perceived from best serving BS. w_q is weight associated with q^{th} bin that can be set by operator to reflect its significance in the planning process, e.g., a bin in dense hotspot area will have higher value of w than a bin in less populated area. While w can be used to incorporate the Quality of Experience (QoE), pricing factors and operators policies etc., here we use a simple function to set the value of w as:

$$w_q = \frac{u_q}{U}, where \quad U = \sum_{q=1}^{Q} u_q \tag{9}$$

where u_q is the expected number of active users in a q^{th} bin, and U is the total number of average active users in the system.

Note that the SIR in (7) is a function of vector of azimuth angles of *all* sectors in the system given by vector ϕ_a^N where $N = |\mathcal{N}|$. Clearly (8) is a large scale non convex optimization problem. Because of having no known polynomial time solution, problems similar to this have been solved using *simulation-heuristics* approach in literature as explained in Section I.

III. Proposed Analytical Solution

In this section we propose a method to obtain an azimuth angle optimization solution without resorting to aforementioned conventionally used *simulation-heuristics*.

We start by underpinning the key factor that makes the problem in (8) too complex to be tackled with analytical approach. That is, instead of looking at the problem in (8) from top-to-bottom perspective where azimuths of all sectors are being tried to be optimised together while considering all the bins in the system, we exploit bottom-to-top perspective that allows one sector azimuth to be optimised at a time. Observing that optimising azimuth in a sector affects only numerator of SIR in (7) for users inside the sector and only denominator of SIR in (7) for user outside the sector, one sector azimuth at time can be optimised while considering only the bins it will affect. That is, for optimising azimuth of a sector, the users inside the sector and those in the projected area of that sector only (as illustrated in Figure 1) need to be considered in the optimisation process. Thus mathematically the problem in (8) can be closely manifested with a decomposed form as:

$$\max_{\phi_a^n \forall \in \phi^{\mathcal{N}}} \left(\frac{\frac{1}{\sum_{\forall q \in \mathcal{Q}^n} w_q} \sum_{\forall q \in \mathcal{Q}^n} \left(w_q \rho_q^n 10^{\tau\left(\varphi_q^n + \left(\phi_q^n - \phi_a^n\right)^2\right)} \right)}{\frac{1}{\sum_{\forall q \in \mathcal{Q}^n_{B_h}} w_q} \sum_{\forall q \in \mathcal{Q}^n_{B_h}} \left(w_q \rho_q^n 10^{\tau\left(\varphi_q^n + \left(\phi_q^n - \phi_a^n\right)^2\right)} \right)} \right) \tag{10}$$

where $\phi^{\mathcal{N}}$ is set of azimuth angles of all sectors in the system, and $\mathcal{Q}_a^n = \mathcal{Q}^n \cup \mathcal{Q}^n_{B_h}$ where $\mathcal{Q}^n$ is set of bins inside the n^{th} sector and $\mathcal{Q}^n_{B_h}$ is set of bins that are not in n^{th} sector but lie in the projection area of n^{th} sector as shown in Figure 1.

The optimisation problem in (10) can be solved with conventional derivative based optimisation methods. For space constraints we have skipped the full derivation and only final result is presented here. By taking the first derivative, tt can be shown that the critical points of the objective function in (10) correspond to azimuth angle given by the following optimality condition:

$$\frac{\sum_{\forall q \in \mathcal{Q}^n} w_q \zeta_q^n (\phi_a^n - \phi_q^n)}{\sum_{\forall q \in \mathcal{Q}^n_{B_h}} w_q \zeta_q^n (\phi_a^n - \phi_q^n)} = \frac{\sum_{\forall q \in \mathcal{Q}^n} w_q \zeta_n^q}{\sum_{\forall q \in \mathcal{Q}^n_{B_h}} w_q \zeta_n^q} \tag{11}$$

where $\zeta_q^n = 10^{-1.2(\phi - \phi_q^n)^2} \times \frac{\sigma_q^n}{\left(d_q^n\right)^\beta}$ effectively denotes the (Received Signal Level) RSL in q^{th} bin from n^{th} sector. The feasibility of obtaining an analytical solution of (10) in from of an easily tractable expression given by (11) now allows us to propose following algorithm for the derived analytical azimuth angle optimisation framework:

Algorithm:

1) Divide the whole area of interest in bins of size that yields a tradeoff between resolution and complexity.
2) Assign weights to each bin to reflect the significance of its coverage in the optimization process, e.g., by incorporating probability of containing active users, targeted QoS, potential revenue etc.
3) Overlay a regular sectorised layout on the gird of bins and calculate the total weight of bins each sector contains.
4) Start with the sector with the highest total weight and set its azimuth angle according to the solution in (11).
5) Continue in descending order of weight of sectors, until all sector azimuth angles are optimised.

IV. Numerical results

In this section we present numerical results by considering a sample network scenario consisting of 3 BS with total 9 sectors, out of which optimisation of one sector is carried out. While these results do not fully predict the full scale system level gain of the proposed solution, their key significance is that they are readily obtainable from the analytical results presented in section III using parameter values given in Table 1. Thus these results fully serve the purpose of validating the potential gain of the proposed solution.

Six sets of numerical results from Figure 2 to 6 are shown to represent different instances of user distributions and corresponding optimal azimuth angles obtained by the proposed solution. In each set, top left figure shows the objective function in (10) plotted as function of range of azimuth angles. Top right figure shows the average RSL perceived by the users inside the sector and average RSL (interference) perceived by the users outside (in projection area) the sector under optimisation. Bottom left figure plots the Left Hand Side (LHS) and Right Hand Side (RHS) of the optimality condition given in (11). Note that according to (11) the intersection of curves representing LHS and RHS provides the maxima (or minima) of the objective function in (10) thus contains the desired solution. This point is highlighted by a cross symbol in left top and bottom figures. Bottom right figure shows the actual user distribution used in the given experiment. Blue arrow in this figure shows the optimal azimuth angle obtained from the proposed solution for that particular user distribution. e.g., in the result set given in Figure 2 , the proposed solution yields optimal azimuth angle of 46 ° that is plotted in the right bottom figure as blue arrow.

Note from the top right figures in all result sets that for different user distributions the objective function has different shapes but has clear optimal point. This highlights the need for a user demography-aware azimuth angle planning scheme. Note from the top right figures that gain of the proposed solution compared to regular azimuth angle (indicated as 90 degree azimuth) also depends on the user distribution. e.g., in scenario represented by result set in Figure 2, compared to regular azimuth angle, proposed solution raises RSL of inside users by $(-63.04 - (-64)) \approx 1$ dB and reduces the interferences received from it by the other sector (outside) users by $(-104.01 - (-106)) \approx 2$dB. This effectively translates to 1dB and 2dB gain in SINR for inside and outside users respectively. Note that, this gain is indicative of optimisation of one sector only. With optimization of all sector azimuths through proposed algorithm, the gain is expected to be even higher. However, implementation of proposed solution in a full scale system level simulator is required to assess this gain which is beyond the scope of this paper and remains topic of future work.

Similarly, from Figure 3 and 4 gains of 0.6dB,1.2dB and 0.2dB,1dB for inside and outside user can be observed.

While the objective function is convex for most demographic scenarios, for certain user demography it can be non-convex as seen in top and bottom right figures of result set Figure 4. However, a very small number of critical points and presence of analytically tractable solution means that the absolute optimal solution of (10) can be easily found. Note that as the user distribution approaches to uniform, the proposed solution converges to the regular azimuth angle. This can be clearly in figure 5 as well as in 6.

V. Conclusions and Future Work

An analytical framework to model cellular system performance as function of sector azimuth angle is presented. An optimisation problem is formulated to optimise azimuth angles and a low complexity scalable solution is then analytically derived. A simple algorithm for pragmatic implementation of proposed solution in real system is also presented. While full scale system level evaluation of proposed solution remains as future work, extensive numerical results obtained provide interesting insights into system behaviour and demonstrate the significant gain of the proposed solution for planning cell azimuth angles without resorting to cumbersome simulation tools and metaheuristics while taking into account variety of factors of real system such as demography, QoS and revenue potentials.

TABLE I
System Level Simulation Parameters

Parameters	Values
System topology	3 BS×3 sector, Frequency Reuse 1
BS Transmission Power	39 dBm
Cell Radius,BS and user height	600, 32 and 1.5 meters respectively
User antenna gain	0 dB (Omini directional)
B_h,B_v	70 °,10 °
$\lambda_v = \lambda_h$	0.5
G_{max},A_{max}	18, 20 dB
Frequency	2 GHz
Pathloss model	3GPP Urban Macro
Bandwidth	5 MHz
Shadowing standard deviation	8 dB

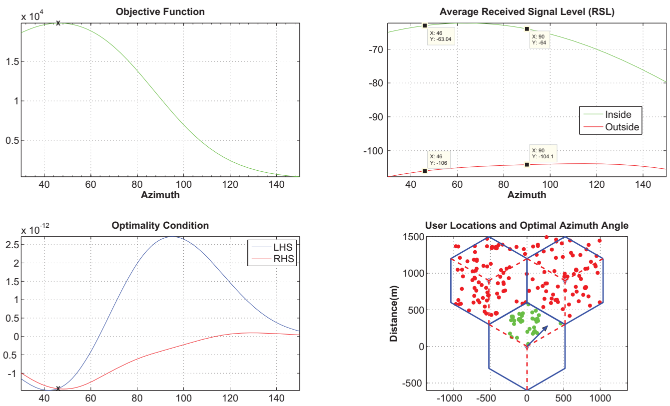

Fig. 2. Optimal azimuth angle and its dependency on demography

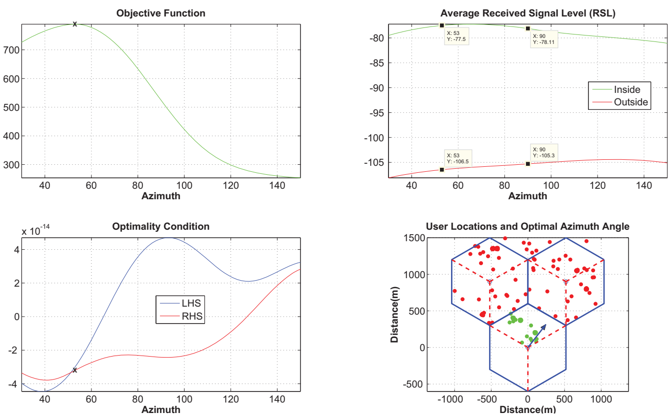

Fig. 3. Optimal azimuth angle and its dependency on demography

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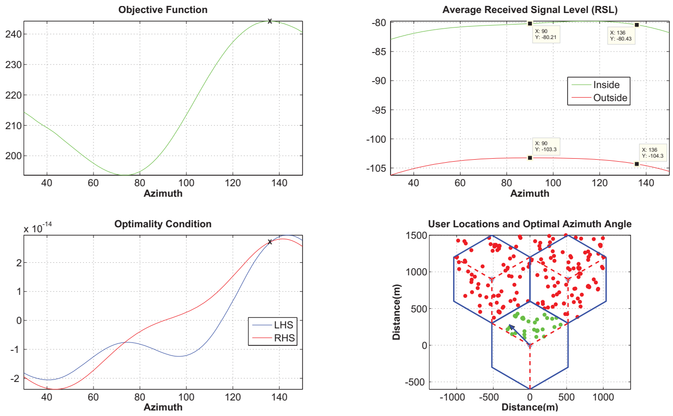

Fig. 4. Possibility of non-convexity of objective function

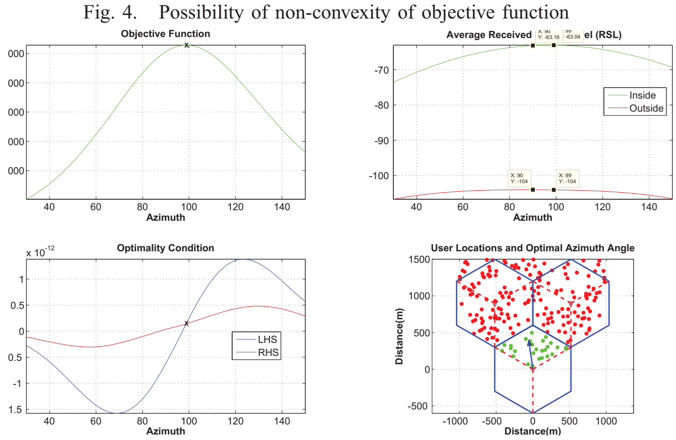

Fig. 5. Convergence to symmetric azimuth in uniform demography

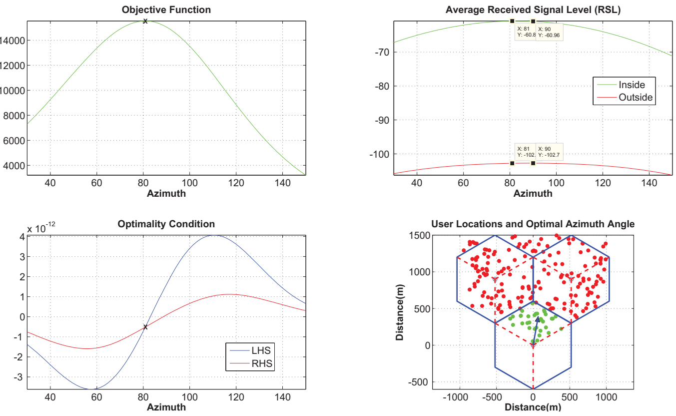

Fig. 6. Convergence to symmetric azimuth in uniform demography

Qatar Foundation). The statements made herein are solely the responsibility of the authors.

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