Impact of Inaccurate User and Base Station Positioning on Autonomous Coverage Estimation

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Abstract—Autonomous monitoring of key performance indicators, which are obtained from measurement reports, is well established as a necessity for enabling self-organising networks. However, these reports are usually tagged with geographical location information which are obtained from positioning techniques and are therefore prone to errors. In this paper, we investigate the impact position estimation errors on the cell coverage probability that can be estimated from autonomous coverage estimation (ACE). We derive novel and accurate expressions of the actual cell coverage probability of such scheme while considering: errors in user equipment (UE) location and; errors in both UE and base station location. We present generic expressions for channel modelled with path-loss and shadowing, and much simplified expressions for the path-loss dominant channel model. Our results reveal that the ACE scheme will be suboptimal as long as there are errors in the reported geographical location information. Hence, appropriate coverage margins must be considered when utilising ACE.

I. INTRODUCTION

Recently, extensive research and standardisation work has focused on the novel paradigm of self-organising network (SON). SON aims at achieving a substantial reduction in capital and operational expenditures (CAPEX & OPEX) by reducing human involvement in network operational tasks, while optimising the networks coverage, capacity and quality of service [1]. In general, SON concept involves the integration of self-configuration, self-optimisation and self-healing functionalities into an automated process requiring minimal manual intervention [1]–[3]. However, these autonomous features cannot be achieved with the current drive test based coverage assessment approach, as it lacks automaticity and therefore results in huge delay and cost.

In order to incorporate SON features, system performance metrics such as coverage, quality of service (QoS), energy efficiency and spectral efficiency need to be monitored and optimised autonomously. This can be achieved by the measurement reports provided by the user equipment (UE) to their serving node. These measurement reports can then be exploited to determine a number of key performance indicators (KPIs), e.g. coverage and service maps, cell boundaries, hotspot locations, congestion indicators and energy consumption indicators, at the base station (BS) autonomously. This leads to a significant saving in time and labor cost on drive test based field measurements. Consequently, autonomous estimation of KPIs can substantially reduce the time frame and cost of the post-deployment optimisation cycle.

These solutions require accurate geographical location information for both the UEs and nodes which may be deployed in an impromptu manner (e.g. Femto cells). Location information can be obtained by using positioning techniques, such as observed time difference of arrival (OTDOA) or assisted global positioning system (A-GPS) [4], [5]. For indoor environments, position estimation techniques based on WLAN, radio frequency identification (RFID) and ultrasonic have been proposed [6]–[9]. All these techniques are prone to errors. For example, the accuracy of A-GPS has been evaluated as 10 m, 10–20 m and 10–100 m for rural, suburban and urban environments, respectively. On the other hand, the average accuracy of indoor techniques is about 2 m, however, they require installing specialised devices [6]–[9].

By exploiting the measurement reports gathered by the UEs and their location information, an autonomous coverage estimation (ACE) can be developed. In such a system, UEs measurement report such as received signal strength (RSS) are tagged with their geographical location information, which are obtained from the position estimation techniques, and sent to their serving BS. The serving BS after retrieving the measurements, further appends its geographical location information and forwards them to a trace collection entity (TCE), which then generates the autonomous coverage map. Since the position estimation techniques are prone to errors, the measurement reports may be tagged to a wrong location.

In this paper, we investigate the impact of inaccurate position estimation on the ACE scheme by deriving its cell coverage probability over the area of interest where the data are gathered from. We build on our earlier work in [10] where we have considered the case with errors only in the reported UE geographical location. In this paper in addition to UE location error, we also consider the case with errors in the estimated location of the BS. We have considered the following channel propagation scenario in our analysis: 1) path-loss dominant channel model and 2) path-loss and shadowing dominant channel model. The rest of this paper is organised as follows: In Section II, we present the framework for ACE. In Section III, we derive the cell coverage probability of the ACE scheme for the channel model with both path-loss and shadowing, while Section IV gives the derivation for the path-loss dominant channel model. In Section V, we present the numerical results which show that our analytical derivations are very accurate. Furthermore, our results show...
that the impact of inaccurate position estimation on the ACE coverage probability becomes more severe as the error in position estimation increases. Finally, Section VI concludes the paper.

II. AUTONOMOUS COVERAGE ESTIMATION FRAMEWORK

We consider an ACE scheme which exploits the measurement reports gathered by the UEs. In such a system, UEs measurement reports are tagged with their geographical location information and sent to their serving BS. The serving BS after retrieving the measurements, further appends its geographical location information and forwards them to a TCE, which can then generate the coverage map. The reported geographical coordinates of the UEs and BSs are obtained from positioning techniques, such as OTDOA or A-GPS [4], [5]. However, these techniques are prone to errors, and, hence the reports may be tagged to a wrong location. In this paper, given a reported UE position, \( o \), with coordinates \((c, d)\), we assume that its actual location is within a circular disc with radius \( r \) which is centered at \( o \), as illustrated in Fig. 1(a). Furthermore, we assume that errors in BS positioning can be resolved such that its displacement from its reported position, \( e \), is known.

For analytical tractability, we consider a single cell deployment scenario where RSS measurement reports are gathered by the UE. The signal propagation model we employ for obtaining the RSS is as follows

\[
P_r(p) = \left( \frac{p}{p_0} \right)^{-\eta} \frac{P_t}{P_l(p_0) - \Phi},
\]

where \( P_r(p) \), \( P_t \) and \( \eta \) denote RSS at distance \( p \) from the BS, transmit power and path-loss exponent, respectively. The parameter \( p_0 \) denotes the reference distance with a known path-loss, \( P_l(p_0) \). The shadowing effect is modeled by the random variable, \( \Phi \), which follows a log-normal distribution such that \( 10\log_{10} \Phi \) follows a zero mean Gaussian distribution with standard deviation \( \sigma \) in dB. The error in coverage estimation as a result of such autonomous scheme is evaluated by assessing the reliability of radio frequency (RF) coverage on the measurement based on the fundamental metric of cell coverage probability.

1) Cell Coverage Probability: In general, the cell coverage probability can be defined as

\[
C = \frac{1}{A} \int \mathbb{P}[P_r(p) \geq \gamma] \, dA,
\]

and can be thought of equivalently as the average fraction of UEs who at any time achieve a target RSS, \( \gamma \), i.e., the average fraction of network area that is in coverage at any time. Hence, given a circular radial distance \( R \) from the BS, we are interested in computing the percentage of area with RSS greater or equal to \( \gamma \).

III. CELL COVERAGE PROBABILITY WITH ACE

Here we consider the scenario where both shadowing and path-loss are the dominant factors in the channel propagation model. The probability that the reported RSS (in dB) at a distance \( p \) from the BS will exceed the threshold \( \gamma \), i.e., \( \mathbb{P}[P_r(p) \geq \gamma] \) can be obtained from [11], [12] as

\[
\mathbb{P}[P_r(p) \geq \gamma] = \frac{1}{2} - \frac{1}{2} \text{erf} \left( a + b \ln \frac{p}{R} \right),
\]

where \( a = \frac{\gamma (\text{dBm}) - P_r(\text{dBm}) + P_l(p_0)(\text{dB}) + 10\eta \log_{10} \frac{R}{p_0}}{\sigma \sqrt{2}} \), and \( b = \frac{10 \eta \log_{10} e}{\sigma \sqrt{2}} \) when there are no errors in UE and BS location information. In the same way, cell coverage probability of the ACE scheme without error in location information can be expressed as

\[
C = \frac{1}{2} - \frac{1}{R^2} \int_0^R \text{erf} \left( a + b \ln \frac{p}{R} \right) dp.
\]

A. UE Geographical Location Information Error

Now we consider the case with errors in the geographical location information reported by the UEs to their serving BS. As stated earlier, the actual location of a UE lies within a circular disc centered at the reported location. Consequently, its actual location with reference to its reported location can be modeled as

\[
\mathbb{P}(\kappa, \phi) = \sqrt{\rho^2 + \kappa^2 - 2 \rho \kappa \cos \phi},
\]

where \( 0 \leq \kappa \leq r \) and \( 0 \leq \phi \leq 2\pi \). The PDF of the distance and direction of the UE’s actual location with respect to its reported position are \( \frac{1}{2\pi} \) and \( \frac{1}{2\pi} \), respectively. Therefore, the modified \( \mathbb{P}[P_r(p) \geq \gamma] \) as a result of the inaccuracies in the UE’s location information can be obtained as

\[
\mathbb{P}[P_r(p) \geq \gamma] = \mathbb{E}_{\kappa, \phi} \left\{ \mathbb{P}[P_r(P(\kappa, \phi)) \geq \gamma) \right\}
\]

\[
= \frac{1}{2\pi r} \int_0^{2\pi} \int_0^r \mathbb{P}[P_r(P(\kappa, \phi)) \geq \gamma) \, d\kappa d\phi,
\]

where \( \mathbb{E} \) is the expectation. This further simplifies as

\[
\mathbb{P}[P_r(p) \geq \gamma] = \frac{1}{2\pi r} \int_0^{2\pi} \int_0^r \frac{1}{2} \text{erf} \left( a + b \ln \frac{p^2 + \kappa^2 - 2p \kappa \cos \phi}{R^2} \right) d\kappa d\phi.
\]

by substituting (3) into (6). Consequently, the actual percentage of the area \( A \) in coverage due to the ACE scheme can be obtained as

![Fig. 1. (a) UE with reported position \( o \), its actual position lies within the circular disc with radius \( r \) centered \( o \). (b) shows the triangle created in (a).](Image 373x740 to 375x741)

![Image 375x741 to 375x741](Image 400x705 to 410x724)

![Image 373x740 to 375x741](Image 400x705 to 410x724)
The actual coverage probability of the ACE scheme over the area \(A\) is given by

\[
C_{ACE} = \frac{1}{A} \int \int_{p} \mathbb{P}[P_r(p) \geq \gamma] dA = \frac{1}{\pi R^2} \int_{0}^{\pi} \int_{0}^{2\pi} \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{b}{2} \sqrt{\frac{1 + \kappa^2 - 2\kappa \cos \phi}{R^2}} \right) \right] d\phi dp.
\]

**B. UE and BS Geographical Location Information Error**

In addition to the UE’s position error, we consider here the scenario where the geographical location information reported by the serving BS to the TCE is displaced at a distance \(c\) from its actual location, as depicted in Fig. 2. Hence, the measurement reports stored in the TCE are also tagged with a wrong BS position, thus resulting in the generation of a wrong coverage map. In order to estimate the actual coverage probability of the ACE scheme over the area \(A\) (circular area) centered at the reported BS position \(X\), we estimate the fraction of the reports that will still be in coverage based on the actual BS position \(X\).

Consider \(R\) as the radius of the area of interest \(A\) centered at \(X\), we can create a virtual representation of \(A\) centered at \(X\) such that both intersects at \(S_1\) and \(S_2\), as shown in Fig. 2. The intersecting points are characterized by the angle, \(\alpha = \pi - \cos^{-1} \left( \frac{c}{R} \right)\). Hence, using this property, we define two regions, \(A_1\) and \(A_2\), which are the shaded and unshaded areas in the area of interest, respectively, and we estimate the actual fraction of UE in coverage based on the actual BS position \(X\).

The distance between the reported UE position in region \(A_1\) and \(A_2\) with respect to the actual BS positions can be expressed as

\[
\tilde{p}_{A_1}(\theta) = \sqrt{R^2 + c^2 - 2Rc \cos \left( \pi - \frac{\sin \theta}{R} \right)}
\]

\[
\tilde{p}_{A_2}(\theta) = \sec \left( \theta - \sin^{-1} \left( \frac{c \sin \theta}{R} \right) \right)
\]

respectively, where \(\pi - \alpha \leq \theta \leq 2\pi - \alpha\) and \(2\pi - \alpha \leq \theta \leq 3\pi - \alpha\) for \(\tilde{p}_{A_1}(\theta)\) and \(\tilde{p}_{A_2}(\theta)\), respectively. Consequently, the actual coverage probability of the ACE scheme over the area \(A\) can be expressed as

\[
C_{ACE} = \frac{2}{\pi R^2} \left( \int_{0}^{\pi} \int_{0}^{2\pi} \tilde{p}_{A_1}(\theta) \mathbb{P}[P_r(p) \geq \gamma] d\theta dp \right) + \left( \int_{0}^{\pi} \int_{0}^{2\pi} \tilde{p}_{A_2}(\theta) \mathbb{P}[P_r(p) \geq \gamma] d\theta dp \right), \quad (11)
\]

when there are errors in both the UE and BS geographical location information. By substituting the expression of \(\mathbb{P}[P_r(p) \geq \gamma]\) in (7) into (11), it is further expressed as (12) given at the top of the next page.

**IV. ACE Coverage Probability: Pathloss Only Channel Model**

Here we consider the scenario where the pathloss is the predominant factor in the channel propagation model. We further assume that the cell coverage radius \(R\) is such that \(R = p_0 \left( \frac{\gamma}{P_{th}(\rho_{th})} \right)^{\eta}\). Hence, for the case with no error in geographical location information and no shadowing, \(\mathbb{P}[P_r(p) \geq \gamma] = 1\), while \(0 \leq p \leq R\). Consequently from (8) and (2), the cell coverage probability over the circular radial distance, \(R, C = 1\), in this case.

**A. UE Geographical Location Information Error**

It can easily be shown that for the case without shadowing and with only UE positioning error, \(\mathbb{P}[P_r(p) \geq \gamma]\) in (6) is equivalent to the fraction of the circular disc area that lies within the cell radius \(R\), as illustrated in Fig. 1. By applying laws of trigonometry, we obtain \(\mathbb{P}[P_r(p) \geq \gamma]\) as follows

\[
\mathbb{P}[P_r(p) \geq \gamma] = \frac{\beta - \sin \beta}{2\pi} + \frac{\theta - \sin \theta}{2\pi} \left( \frac{R}{r} \right)^2, \quad (13)
\]

where

\[
\beta(p) = 2 \cos^{-1} \left[ \frac{\kappa + \sqrt{\kappa^2 + \frac{2\kappa - R^2}{2\kappa}}}{2\kappa} \right], \quad 0 \leq p \leq R.
\]

Hence, the cell coverage probability over the area \(A\) and as a result of the ACE scheme can be obtained according to (8) as

\[
C_{ACE} = \frac{1}{A} \int \mathbb{P}[P_r(p) \geq \gamma] dA = \frac{1}{\pi R^2} \int_{0}^{2\pi} \int_{0}^{p} \left( \frac{\beta - \sin \beta}{2\pi} + \frac{\theta - \sin \theta}{2\pi} \left( \frac{R}{r} \right)^2 \right) dp.
\]

for the case without shadowing but with error in UE positioning.

**B. UE and BS Geographical Location Information Error**

Following a similar approach with the shadowing case, we derive the cell coverage probability for the case with errors in both the UE and BS geographical location information. The cell coverage probability of the ACE for the case with pathloss as the dominant factor in the channel propagation model can also be expressed as in (11), but with \(\mathbb{P}[P_r(p) \geq \gamma]\) defined in (13). We thus arrive at (15) given at the top of the next page.

**V. Numerical Results**

In this section, we present numerical results in order to verify the accuracy of the proposed analytical methodology against simulations, as well as to show the impact of errors in reported geographical location information on the actual coverage estimated by the ACE scheme. We consider a single cell.
\[
C_{ACE} = \frac{2}{\pi R^2} \left( \int_0^{\pi - \alpha} \int_0^{\rho_{\Phi}(\theta)} \int_0^{\pi} \int_0^{\rho_{\Phi}(\theta)} p \left[ \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{a + b \ln p^2 + \kappa^2 - \frac{2p\cos \phi}{R}}{R^2} \right) \right] d\phi d\rho d\theta \right) \\
+ \int_0^{\alpha} \int_0^{\rho_{\Phi}(\theta)} \int_0^{\pi} \int_0^{\rho_{\Phi}(\theta)} p \left[ \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{a + b \ln p^2 + \kappa^2 - \frac{2p\cos \phi}{R}}{R^2} \right) \right] d\phi d\rho d\theta \right) .
\] 
(12)

\[
C_{ACE} = \frac{2}{\pi R^2} \left( \int_0^{\pi - \alpha} \int_0^{\rho_{\Phi}(\theta)} \int_0^{\pi} \int_0^{\rho_{\Phi}(\theta)} p \left( \frac{\beta - \sin \beta}{2\pi} + \frac{\theta - \sin \theta}{2\pi} \right) d\phi d\rho d\theta + \int_0^{\alpha} \int_0^{\rho_{\Phi}(\theta)} \int_0^{\pi} \int_0^{\rho_{\Phi}(\theta)} p \left( \frac{\beta - \sin \beta}{2\pi} + \frac{\theta - \sin \theta}{2\pi} \right) d\phi d\rho d\theta \right) .
\] 
(15)

### Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>(\sigma)</td>
<td>7, 9, 12 dB</td>
</tr>
<tr>
<td>Path Loss Exponent</td>
<td>(\eta)</td>
<td>3.5</td>
</tr>
<tr>
<td>Reference Distance</td>
<td>(p_0)</td>
<td>1 m</td>
</tr>
<tr>
<td>Path Loss at (p_0)</td>
<td>(P_{L}(p_0))</td>
<td>34.5 dB</td>
</tr>
<tr>
<td>Power Transmitted</td>
<td>(P_t)</td>
<td>46 dBm</td>
</tr>
<tr>
<td>Threshold</td>
<td>(\gamma)</td>
<td>-84.5 dBm</td>
</tr>
<tr>
<td>UE position error</td>
<td>(r)</td>
<td>10 – 100 m</td>
</tr>
<tr>
<td>BS position error</td>
<td>(e)</td>
<td>20 m</td>
</tr>
</tbody>
</table>

deployment with the parameters specified in Table I and we estimate the cell coverage probability over a circular coverage region of area \(\pi R^2\), where \(R = p_0 \left( \frac{\gamma P_{L}(p_0)}{P_t} \right)^{\frac{1}{2}} \approx 553.1681\) m.

As far as simulations are concerned, we used the following methodology for the case with errors in geographical location information reported by the UE.

1) 1,000,000 UE are distributed following a uniform distribution over the circular region of area \(\pi R^2\) and their positions are taken as the reported position.

2) The actual position of the \(i^{th}\) UE with coordinates \((c_i, d_i)\) is generated as \((c_i + r \sqrt{\pi} \cos(2\pi v_i), d_i + r \sqrt{\pi} \sin(2\pi v_i))\), where \(v_i\) and \(u_i\) are pseudo independent numbers uniformly distributed in \([0, 1]\).

3) The RSS at the generated actual position \(P_i(\bar{p})\), which is at the distance \(\rho\) from the BS, is estimated according to (1), where \(\Phi = 1\) for the path-loss dominant channel model.

4) The cell coverage probability achieved by the ACE scheme is then evaluated as the percentage of UE with \(P_i(\bar{p}) \geq \gamma\).

For the case with error in reported BS geographical location information, step 3 is changed as follows to incorporate this error.

- Given the BS with reported coordinates \((x, y)\), its actual coordinates is obtained as \((x + e, y)\).
- The RSS is estimated according to (1) based on the distance between the actual BS and UT positions, where \(\Phi = 1\) for the path-loss only channel model.

In Figs. 3 and 4, we validate cell coverage probability expressions that were derived for the ACE scheme, for both the case with errors in reported UE geographical location information, and the case with errors in both reported UE and BS geographical location information. In Fig. 3, we compare our analytical results on the actual cell coverage probability of the ACE scheme with the simulated results, for the case when path-loss and shadowing are the dominant factors in the signal propagation model. Whereas, a comparison for the case with path-loss as the dominant factor is presented in Fig. 4. We note that in both figures, our analytical results closely matches with the simulation. The results in Figs. 3 and 4 further show that the estimated cell coverage probability as measured by the ACE scheme decreases as the UT position error increases. Furthermore, having errors in BS location information further degrades the performance of the ACE scheme.

In Fig. 5, we plot the coverage estimation error as a result of using the ACE scheme, \(D_{ACE}\), against the UE position error, for shadowing standard deviation \(\sigma = 7, 9, 12\) dB and BS position error \(e = 0, 20\) m. We define the coverage estimation error, \(D_{ACE}\), as

\[
D_{ACE} = \frac{C_{ACE} - C}{C} \times 100\% ,
\] 
(16)

where for the shadowing and path-loss dominant channel model, \(C_{ACE}\) is given in (8) and (12), for the case with errors in reported UE geographical location information and for the case with errors in both the reported UE and BS geographical location information, respectively, while \(C\) is given in (4) for the case without errors in reported geographical location information. In addition, \(C_{ACE}\) for the path-loss dominant channel model is given in (14) and (15) accordingly and \(C = 1\) in this case. Fig. 5 shows that the performance of the ACE scheme in estimating the actual coverage deprecates as the error in UE position increases. Furthermore, it can be observed that the performance of the ACE scheme becomes more degraded as the shadowing standard deviation \(\sigma\) reduces. This implies that errors in UE and BS position estimation are less severe on the coverage as \(\sigma\) increases. The reason for this is that increasing \(\sigma\) introduces more randomness to the received signal; hence randomness created by the UE positioning error would have more impact on a lower \(\sigma\).

### VI. Conclusions

In this paper, we have investigated the impact of inaccurate geographical location information on the coverage probability
estimation through an autonomous coverage estimation (ACE) scheme. We have derived the expression of the actual cell coverage probability that can be obtained from such scheme while considering: errors in user equipment (UE) geographical location information and; errors in both UE and base station (BS) geographical location information. The accuracy of the derived expressions has been shown through numerical results for a range of UE and BS positioning errors. The performance of the ACE scheme will be suboptimal as long as there are errors in the reported geographical location information. Hence, appropriate correction factors, that can be calculated using proposed model, must be used while utilising such ACE scheme.

Fig. 3. The coverage probability with ACE scheme as a function of UE position error radius for the pathloss dominant channel propagation model. BS position error $e = 20$ m in (15).

Fig. 4. The coverage probability with ACE scheme as a function of UE position error radius when $\sigma = 9$ dB. BS position error $e = 20$ m in (12).

Fig. 5. Cell coverage degradation with ASE

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